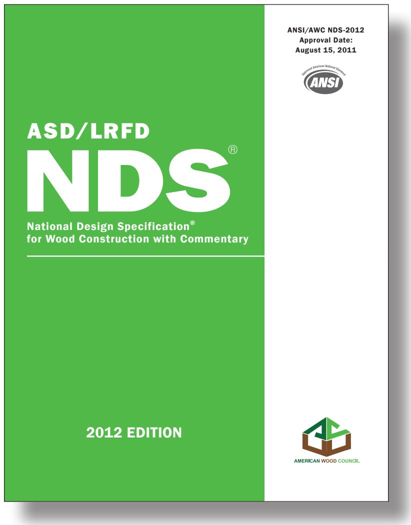
TECHNICAL REPORT 12

General Dowel Equations for Calculating Lateral Connection Values



AMERICAN WOOD COUNCIL

The American Wood Council (AWC) is the voice of North American traditional and engineered wood products. From a renewable resource that absorbs and sequesters carbon, the wood products industry makes products that are essential to everyday life. AWC's engineers, technologists, scientists, and building code experts develop state-of-the-art engineering data, technology, and standards on structural wood products for use by design professionals, building officials, and wood products manufacturers to assure the safe and efficient design and use of wood structural components.

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General Dowel Equations for Calculating Lateral Connection Values

AMERICAN WOOD COUNCIL

Table of Contents

List of Tables

List of Figures

PART 1. General Dowel Equations

1.1 Introduction

A generalized form of the *National Design Specification® (NDS®) for Wood Construction* (AWC, 2012) yield limit equations applicable to dowel-type fastener connections between members of solid cross section is provided in Table 1-1 of this report. Equations are also provided for connections with side or main members of hollow cross section, in Tables 1-2 and 1-3, respectively. The variables for these tables are defined in Section 1.6. These general dowel equations are applicable to NDS connection conditions, allow for evaluation of connections with gaps between connected members, and include separate equation variables to better account for varying bearing and moment resistances of fasteners having more than one diameter along their length (such as threaded fasteners).

1.2 Lateral Connection Values

Yield limit equations provided herein apply to calculation of lateral values for single fastener connections between wood-based members and connections of wood-based members to steel and concrete/masonry components. Design criteria for dowel type fastener connections are provided in the *NDS for Wood Construction* and include the following considerations:

- fastener design limit states (e.g. tension, bearing, and shear);
- fastener spacing, edge, and end distance;
- connection fabrication and tolerances;
- connection geometry;
- multiple fasteners and group action;
- member strength at the connection;
- adjustments for end use (e.g. load duration and wet service) and fastener type (e.g. drift bolts and drift pins).
- member bearing strengths;
- fastener bending strengths;
- reduction factors, R_d, to adjust reference 5% offset yield connection values, P, to reference allowable stress design values, Z.

1.3 Reference Design Value

The reference design value, Z, is the minimum of the calculated, P/R_d, for all yield modes as follows:

$$Z = Minmum\ value\ of\ \left(\frac{P}{R_d}\right) \tag{1}$$

where:

Z = Reference lateral design value

P = Reference 5% offset yield value

R_d = NDS reduction term for dowel type fastener connections, see NDS Table 11.3.1B

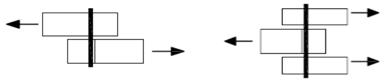
1.4 Connections with Members of Solid Cross Section

Equations in Table 1-1 are applicable for calculation of reference 5% offset yield, P, for dowel type fastener connections where connected members are of solid cross section. The reference 5% offset yield, P, is calculated for each applicable connection yield mode, as depicted in Table 1-1. Example applications of equations in Table 1-1 are provided in Part 3 of this report.

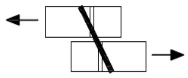
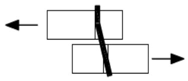
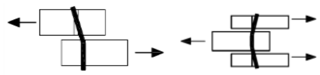
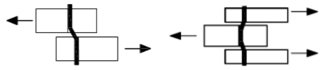
1.5 Connections with Members of Hollow Cross Section

Equations in Table 1-2 are applicable for calculation of reference 5% offset yield, P, for dowel type fastener connections where the main member is of solid cross section and the side member(s) have a hollow cross section. Equations in Table 1-3 are applicable for calculation of reference 5% offset yield, P, for dowel type fastener connections where the main member has a hollow cross section and the side member(s) is/are of solid cross section. The equations in Tables 1-2 and 1-3 are based on the assumption that the dowel penetrates both walls of the hollow member and the thickness of each wall through which the dowel penetrates, t_{ws} or t_{wm}, is the same. Connection yield modes are depicted in Tables 1-2 and 1-3. Example applications of equations in Table 1-3 are provided in Part 3 of this report.

Table 1-1 General Dowel Equations for Solid Cross Section Members[2]

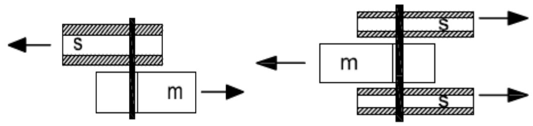
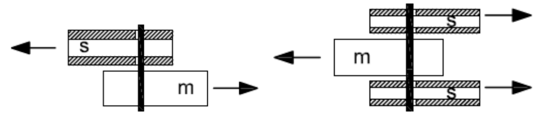
Yield Mode	Single Shear	Double Shear	Description
I_m	$P = q_m L_m$	$P = q_m L_m$	
I_s	$P = q_s L_s$	$P = 2 q_s L_s$	
II-IV	$P = \dfrac{-B + \sqrt{B^2 - 4AC}}{2A}$	$P = \dfrac{-B + \sqrt{B^2 - 4AC}}{A}$	General equation for member bearing and dowel yielding

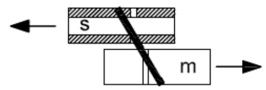
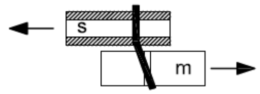
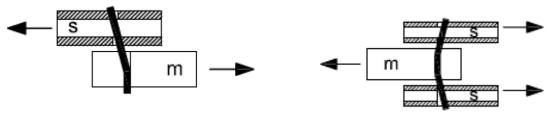
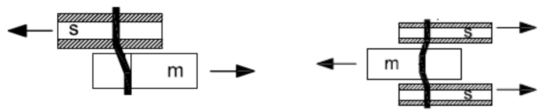
Inputs A, B, & C for Yield Modes II-IV

Yield Mode	A	B	C	Description
II[1]	$A = \dfrac{1}{4q_s} + \dfrac{1}{4q_m}$	$B = \dfrac{L_s}{2} + g + \dfrac{L_m}{2}$	$C = -\dfrac{q_s L_s^2}{4} - \dfrac{q_m L_m^2}{4}$	
III_m[1]	$A = \dfrac{1}{2q_s} + \dfrac{1}{4q_m}$	$B = g + \dfrac{L_m}{2}$	$C = -M_s - \dfrac{q_m L_m^2}{4}$	
III_s	$A = \dfrac{1}{4q_s} + \dfrac{1}{2q_m}$	$B = \dfrac{L_s}{2} + g$	$C = -\dfrac{q_s L_s^2}{4} - M_m$	
IV	$A = \dfrac{1}{2q_s} + \dfrac{1}{2q_m}$	$B = g$	$C = -M_s - M_m$	

[1]Yield Modes II and III_m do not apply for double shear connections.
[2]See Section 1.6 for notation.

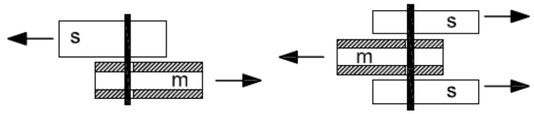
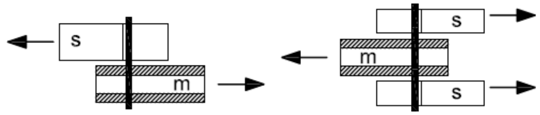
Table 1-2 General Dowel Equations for Solid Cross-Section Main Member and Hollow Cross Section Side Member(s)²

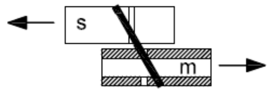
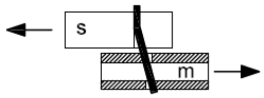
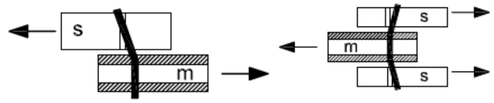
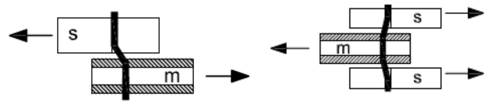
Yield Mode	Single Shear	Double Shear	Description
I_m	$P = q_m L_m$	$P = q_m L_m$	
I_s	$P = 2 q_s t_{ws}$	$P = 4 q_s t_{ws}$	
II-IV	$P = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$	$P = \frac{-B + \sqrt{B^2 - 4AC}}{A}$	General equation for member bearing and dowel yielding

	Inputs A, B, & C for Yield Modes II-IV			
II[1]	$A = \frac{1}{4q_s} + \frac{1}{4q_m}$	$B = t_{ws} + v_s + g + \frac{L_m}{2}$	$C = -q_s t_{ws}(t_{ws} + v_s) - \frac{q_m L_m^2}{4}$	
III_m[1]	$A = \frac{1}{2q_s} + \frac{1}{4q_m}$	$B = g + \frac{L_m}{2}$	$C = -M_s - \frac{q_m L_m^2}{4}$	
III_s	$A = \frac{1}{4q_s} + \frac{1}{2q_m}$	$B = t_{ws} + v_s + g$	$C = -q_s t_{ws}(t_{ws} + v_s) - M_m$	
IV	$A = \frac{1}{2q_s} + \frac{1}{2q_m}$	$B = g$	$C = -M_s - M_m$	

[1]Yield Modes II and III_m do not apply for double shear connections.
[2]See Section 1.6 for notation.

Table 1-3 General Dowel Equations for Hollow Cross Section Main Member and Solid Cross Section Side Member(s)²

Yield Mode	Single Shear	Double Shear	Description
I_m	$P = 2q_m t_{wm}$	$P = 2q_m t_{wm}$	
I_s	$P = q_s L_s$	$P = 2\,q_s L_s$	
II-IV	$P = \dfrac{-B + \sqrt{B^2 - 4AC}}{2A}$	$P = \dfrac{-B + \sqrt{B^2 - 4AC}}{A}$	General equation for member bearing and dowel yielding

	Inputs A, B, & C for Yield Modes II-IV			
II^1	$A = \dfrac{1}{4q_s} + \dfrac{1}{4q_m}$	$B = t_{wm} + v_m + g + \dfrac{L_s}{2}$	$C = -q_m t_{wm}(t_{wm} + v_m) - \dfrac{q_s L_s^2}{4}$	
$III_m{}^1$	$A = \dfrac{1}{2q_s} + \dfrac{1}{4q_m}$	$B = g + t_{wm} + v_m$	$C = -M_s - q_m t_{wm}(t_{wm} + v_m)$	
III_s	$A = \dfrac{1}{4q_s} + \dfrac{1}{2q_m}$	$B = \dfrac{L_s}{2} + g$	$C = -\dfrac{q_s L_s^2}{4} - M_m$	
IV	$A = \dfrac{1}{2q_s} + \dfrac{1}{2q_m}$	$B = g$	$C = -M_s - M_m$	

[1] Yield Modes II and III_m do not apply for double shear connections.
[2] See Section 1.6 for notation.

1.6 Notation

D = dowel diameter, in.
D_s = dowel diameter at maximum stress in side member, in.
D_m = dowel diameter at maximum stress in main member, in.
D_r = fastener root diameter, in.
E = length of tapered dowel tip, in.
F_b = dowel bending strength, psi
F_e = dowel bearing strength, psi
$F_{e\parallel}$ = dowel bearing strength parallel to grain, psi
$F_{e\perp}$ = dowel bearing strength perpendicular to grain, psi
F_{es} = side member dowel bearing strength, psi
F_{em} = main member dowel bearing strength, psi
F_u = tensile strength, psi
F_y = tensile yield strength, psi
G = specific gravity
g = gap between members, in.
L_s = side member dowel-bearing length, in.
L_m = main member dowel-bearing length, in.
M_{bm} = main member dowel bearing maximum moment, in.-lbs
M_{bs} = side member dowel bearing maximum moment, in.-lbs
M_{ds} = side member dowel moment resistance, $F_b(D_s^3/6)$, in.-lbs
M_{dm} = main member dowel moment resistance, $F_b(D_m^3/6)$, in.-lbs
M_s = maximum moment developed in side member at x_s, in.-lbs
M_m = maximum moment developed in main member at x, in.-lbs
P = reference 5% offset yield lateral connection value for a single fastener connection, lbs
p = dowel penetration, in.
q_s = side member dowel-bearing resistance, $F_{es}D$, lbs./in.
q_m = main member dowel-bearing resistance, $F_{em}D$, lbs./in.
t = thickness, in.
t_{ws} = wall thickness in hollow side member, in.
t_{wm} = wall thickness in hollow main member, in.
v_s = length of void space along the axis of the dowel in hollow side member, in.
v_m = length of void space along the axis of the dowel in hollow main member, in.
x = distance from shear plane to maximum moment, in.
R_d = NDS reduction term for dowel type fasteners
Z = reference lateral design value for a single fastener connection, lbs

PART 2. Equation Derivation

2.1 Introduction

The yield model used to develop the general dowel equations considers effects of dowel moment resistance and dowel bearing resistance on a connection's lateral strength. Based on the European Yield Model (Soltis 1991), connection strength is assumed to be reached when: (1) compressive strength of the member beneath the dowel is exceeded; or (2) one or more plastic hinges forms in the dowel. Behavior of the connection is assumed to be in accordance with yield modes depicted in Tables 1-1, 1-2 and 1-3. Dowel loading is assumed to be uniformly distributed and perpendicular to the axis of the dowel (e.g. ideally plastic deformation). Each yield mode addresses a specific loading condition on the dowel such that the dowel will remain in static equilibrium. General dowel equations can be obtained by considering equilibrium of forces within a connection exhibiting behavior in accordance with yield modes I – IV as shown in Tables 1-1, 1-2 and 1-3, and Figures 2-1 and 2-2. Applying this concept, a free-body diagram for each yield mode can be drawn, and principles of statics can be used to develop the general dowel equations.

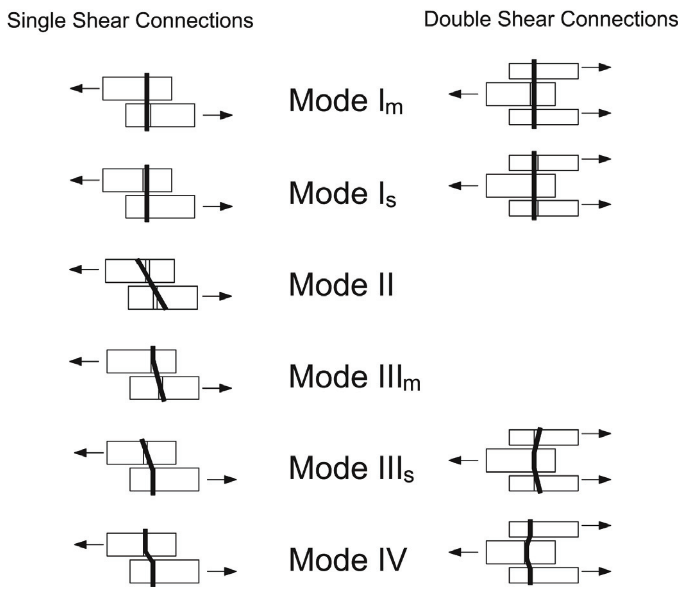
2.1.1 End Fixity and Friction

Potential increased strength due to effects of end fixity, tension forces in the fastener, and friction between members are conservatively ignored. End fixity is the resistance to rotation provided at the end(s) of the dowel, such as under a nail head or bolt nut and washer. The strength contribution of end fixity is difficult to predict and may be dependent on several factors including load level, fastener type and installation, washer or fastener head size (e.g. nail or screw head size), tensile forces that may develop in the fastener under lateral loading, and amount of dimensional change in connected members. Frictional resistance to slipping of connection members is not typically addressed in wood connection design because the amount of frictional force is difficult to predict and in many instances may not exist as wood shrinks or the connection relaxes.

2.1.2 Predicted Yield Mode

Predicted yield modes for single fastener connections are in agreement with yield modes observed from connection tests where end and edge distance, member cross section, and fastener size are adequate to ensure development of yielding by precluding occurrence of premature wood failures such as those due to splitting, formation of a wood shear plug, fastener failure or fastener withdrawal. These and other failure limit states are not specifically addressed by the yield mode equations.

Figure 2-1 Connection Yield Modes – Solid Cross Section Members Section Members



"m" denotes main member, "s" denotes side member

2.2 Equation Derivation for Connections with Members of Solid Cross Section

2.2.1 Mode I

Yield modes I_m and I_s model connections limited by uniform bearing in the main and side member(s). Figure 2-3, Case A, shows that the maximum value for shear, P, is determined by the following equations:

$$P = q_m L_m \tag{2}$$

$$P = q_s L_s \tag{3}$$

Similarly, considering the geometry of a double shear connection, the maximum value for shear, P, is determined by:

$$P = q_m L_m \tag{2}$$

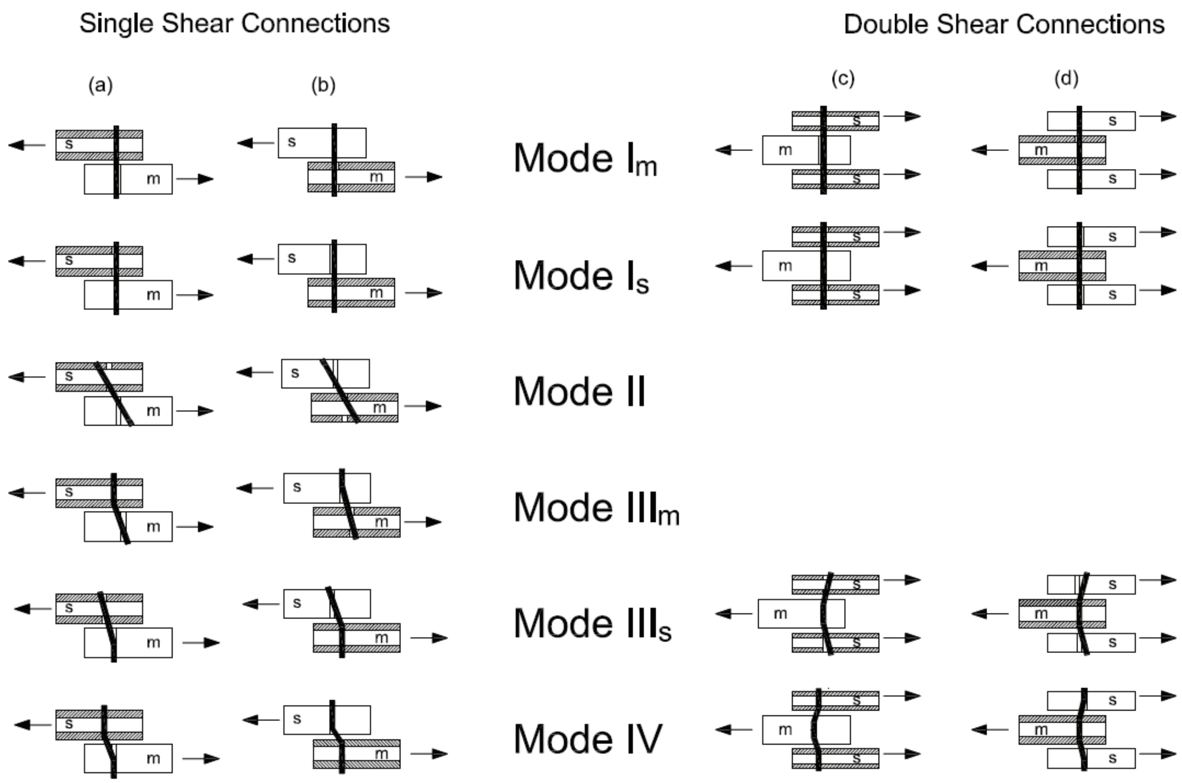
$$P = 2q_s L_s \tag{4}$$

2.2.2 Modes II-IV

For modes II-IV, considering conditions where limiting or maximum moments act on the dowel simplifies equation derivation. These basic conditions are shown in Figure 2-3. In Case B, maximum moment is based on dowel bearing. In Case C, maximum moment is based on dowel bending. Maximum moments for both cases occur at points of zero shear. Maximum moment due to dowel bearing, shown in Case B of Figure 2-3, represents a load condition where the dowel's moment capacity is sufficiently large to prevent yielding of the dowel (dowel bending). For calculation purposes, let q define member bearing resistance (lb/in.) ($q = F_e D_b$), L, define member bearing length (in.), and let x represent the location of zero shear ($x = L - 2a$). The resulting maximum moment due to dowel bearing, M_b, equals qa^2. Recognizing that $a = (L - x)/2$ and $x = P/q$ and substituting results in $M_b = (q/4)(L - P/q)^2$ for single shear connections.

Subscripts *s* and *m* indicate side and main member in the following equations for maximum moment due to dowel bearing:

Figure 2-2 Connection Yield Modes – Solid and Hollow Cross Section Members



"m" denotes main member, "s" denotes side member

Single Shear:

$$M_{bs} = \frac{q_s}{4}\left(L_s - \frac{P}{q_s}\right)^2 \tag{5}$$

$$M_{bm} = \frac{q_m}{4}\left(L_m - \frac{P}{q_m}\right)^2 \tag{6}$$

Double Shear:

$$M_{bs} = \frac{q_s}{4}\left(L_s - \frac{P}{2q_s}\right)^2 \tag{7}$$

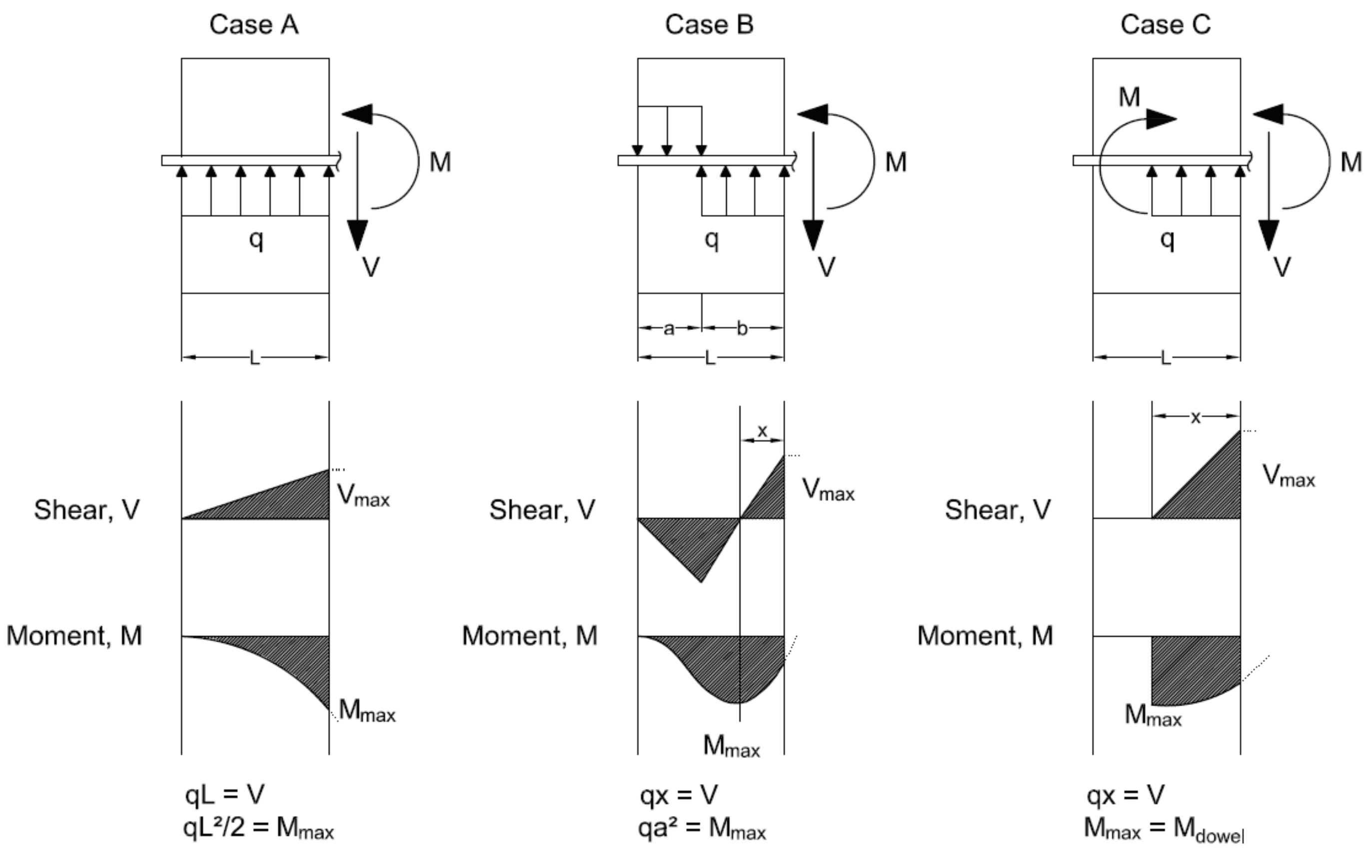
Maximum moment due to dowel bending is shown in Case C of Figure 2-3. In this case, maximum moment is limited to the moment provided by the dowel in bending which is represented by a concentrated moment acting at the point of zero shear (x = P/q). Moment resistance of the dowel, assuming ideally plastic behavior, is expressed as follows:

$$M_{ds} = \frac{F_b D_s^3}{6} \tag{8}$$

$$M_{dm} = \frac{F_b D_m^3}{6} \tag{9}$$

Assumed solid section loading conditions for each of the yield modes I-IV is provided in Figure 2-4. Each mode consists of an interaction of dowel bearing and dowel bending as shown in Figure 2-3. Considering equilibrium of the dowel for each particular yield mode (by summing moments about a fixed point on the dowel), and using relationships for maximum moment defined above, characteristic equations for the maximum load, P, can be determined as follows:

Figure 2-3 General Conditions of Dowel Loading – Solid Members



Single Shear:

$$P^2\left(\frac{1}{2q_s}+\frac{1}{2q_m}\right)+P\,g-(M_s+M_m) = 0 \quad (10)$$

Double Shear:

$$\frac{P^2}{4}\left(\frac{1}{2q_s}+\frac{1}{2q_m}\right)+\frac{P\,g}{2}-(M_s+M_m) = 0 \quad (11)$$

where:

M_s = maximum moment developed in the side member at x_s

M_m = maximum moment developed in the main member at x_m

g = gap distance (assumed to be equal for double shear connections)

Variables M_s and M_m represent values of maximum moment due to dowel bearing or dowel bending depending on the mode being considered. An example single shear connection with assumed loading for yield mode II is provided in Figure 2-5. Shear and moment diagrams are also provided. As shown in Figure 2-5, maximum moments, M_s and M_m for mode II occur at distance x_s and x_m from connected faces, and are based on moments M_{bs} and M_{bm} due to dowel bearing.

Derivation of the general dowel equations assumes that critical stresses in the dowel occur at locations of maximum induced moment. This is appropriate for dowels having a constant diameter in the side and main member. Dowel diameters in the side and main member, however, do not need to be equal. For connections where dowel diameter is not constant within a member, it is conservative to assume that the least diameter occurs at the location of maximum moment. Alternatively, critical stress in the dowel can be determined by considering the applicable moment and dowel section properties along the length of the dowel.

Yield modes II and III_m are not possible for double shear connections, as the assumed symmetry of double shear connections does not permit these modes to occur.

2.2.3 Mode II

Yield Mode II models a connection limited by dowel bearing in the side and main members. The maximum induced moment was previously determined as follows:

Single Shear:

$$M_{bs} = \frac{q_s}{4}\left(L_s-\frac{P}{q_s}\right)^2 \quad (5)$$

$$M_{bm} = \frac{q_m}{4}\left(L_m-\frac{P}{q_m}\right)^2 \quad (6)$$

Substituting M_{bs} and M_{bm} into the characteristic equation for M_s and M_m results in the following quadratic equation expressed in terms of known properties for dowel-bearing resistance (q_s and q_m), dowel-bearing length (L_s and L_m), and the gap between members (g), and a single unknown variable, P.

Single Shear:

$$P^2\left(\frac{1}{4q_s}+\frac{1}{4q_m}\right)+P\left(\frac{L_s}{2}+g+\frac{L_m}{2}\right)-\left(\frac{q_sL_s^2}{4}+\frac{q_mL_m^2}{4}\right)=0 \quad (12)$$

2.2.4 Mode III_m

Mode III_m models a connection limited by dowel bearing in the main member and dowel bending in the side member. The maximum load, P, is determined by solving the characteristic equation for P:

Single Shear:

$$M_{bm} = \frac{q_m}{4}\left(L_m-\frac{P}{q_m}\right)^2 \quad (13)$$

and:

M_{ds} = side member dowel moment resistance

Substituting M_{ds} and M_{bm} into the characteristic equation for M_s and M_m results in the following quadratic equation expressed in terms of known properties for dowel-bearing resistance (q_s and q_m), dowel-bearing length (L_m and L_{ds}), and the gap between members (g), and a single unknown variable, P.

Single Shear:

$$P^2\left(\frac{1}{2q_s}+\frac{1}{4q_m}\right)+P\left(g+\frac{L_m}{2}\right)-\left(M_{ds}+\frac{q_mL_m^2}{4}\right)=0 \quad (14)$$

2.2.5 Mode III_s

Mode III_s models a connection limited by dowel bearing in the side member(s) and dowel bending in the main member. The maximum load, P, is determined by solving the characteristic equation for P.

Single Shear:

$$M_{bs} = \frac{q_s}{4}\left(L_s - \frac{P}{q_s}\right)^2 \quad (5)$$

and:

M_m = dowel bending moment in the main member

Double Shear:

$$M_{bs} = \frac{q_s}{4}\left(L_s - \frac{P}{2q_s}\right)^2 \quad (7)$$

and:

M_{dm} = main member dowel moment resistance

Substituting M_{bs} and M_{dm} into the characteristic equations (equations 10 and 11) for M_s and M_m results in the following quadratic equation(s) expressed in terms of known properties of q_s, q_m, L_s, $M_{m,}$ and g, and a single unknown variable, P.

Single Shear:

$$P^2\left(\frac{1}{4q_s}+\frac{1}{2q_m}\right)+P\left(\frac{L_s}{2}+g\right)-\left(\frac{q_sL_s^2}{4}+M_{dm}\right)=0 \quad (15)$$

Double Shear:

$$\frac{P^2}{4}\left(\frac{1}{4q_s}+\frac{1}{2q_m}\right)+\frac{P}{2}\left(\frac{L_s}{2}+g\right)-\left(\frac{q_sL_s^2}{4}+M_m\right)=0 \quad (16)$$

2.2.6 Mode IV

Mode IV models a connection limited by dowel bending in the main and side member(s). The maximum load, P, is determined by solving the characteristic equation for P:

where:

M_{ds} = side member dowel moment resistance

M_{dm} = main member dowel moment resistance

Substituting M_{ds} and M_{dm} into the characteristic equations (equations 10 and 11) for M_s and M_m results in the following quadratic equations expressed in terms of known properties for dowel-bearing resistance (q_s and q_m), dowel-bearing length (M_s and M_m), and the gap between members (g), and a single unknown variable, P.

Single Shear:

$$P^2\left(\frac{1}{2q_s}+\frac{1}{2q_m}\right)+Pg-\left(M_{ds}+M_{dm}\right)=0 \quad (17)$$

Double Shear:

$$\frac{P^2}{4}\left(\frac{1}{2q_s}+\frac{1}{2q_m}\right)+\frac{Pg}{2}-\left(M_{ds}+M_{dm}\right)=0 \quad (18)$$

Figure 2-4 Connection Yield Modes Assumed Loading – Solid Members

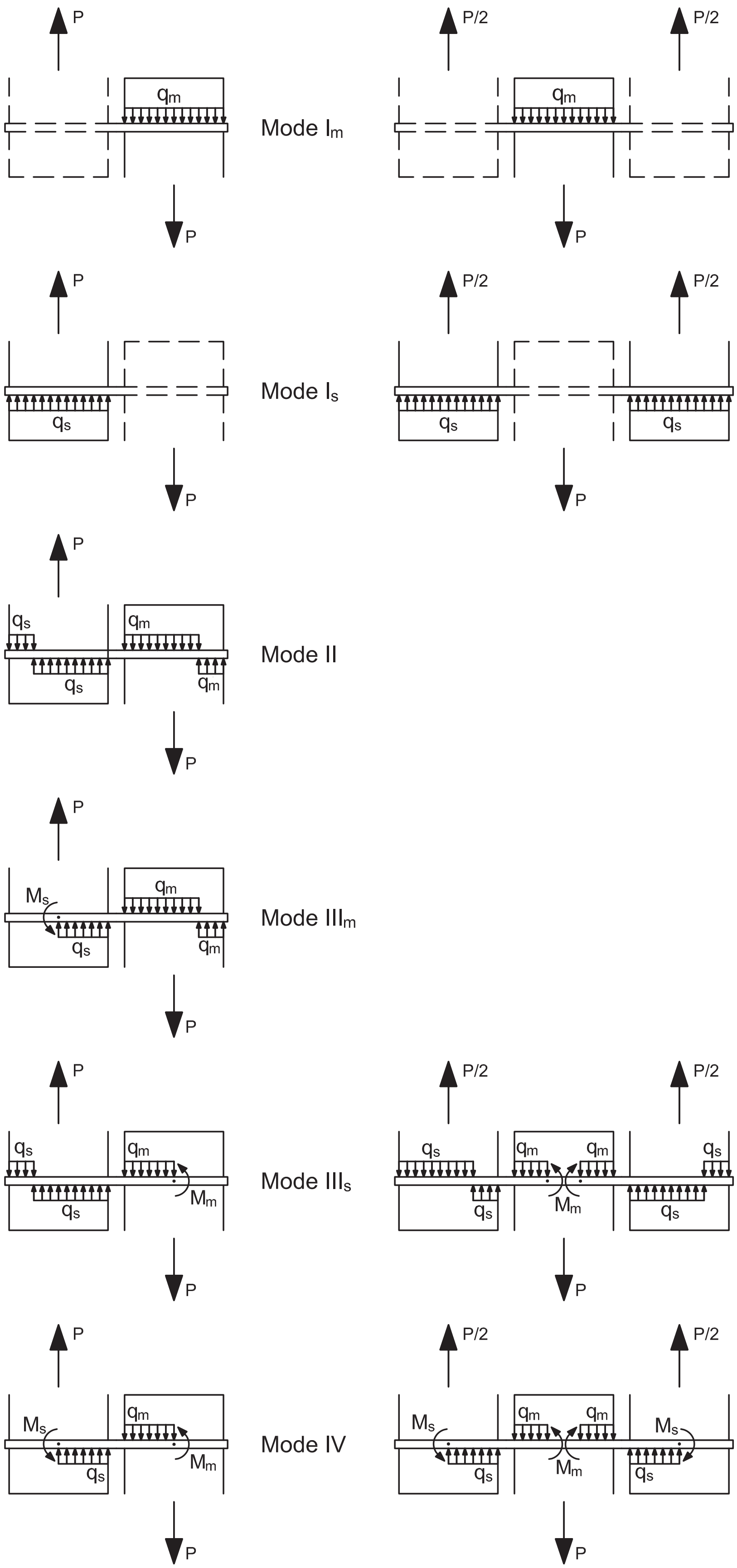
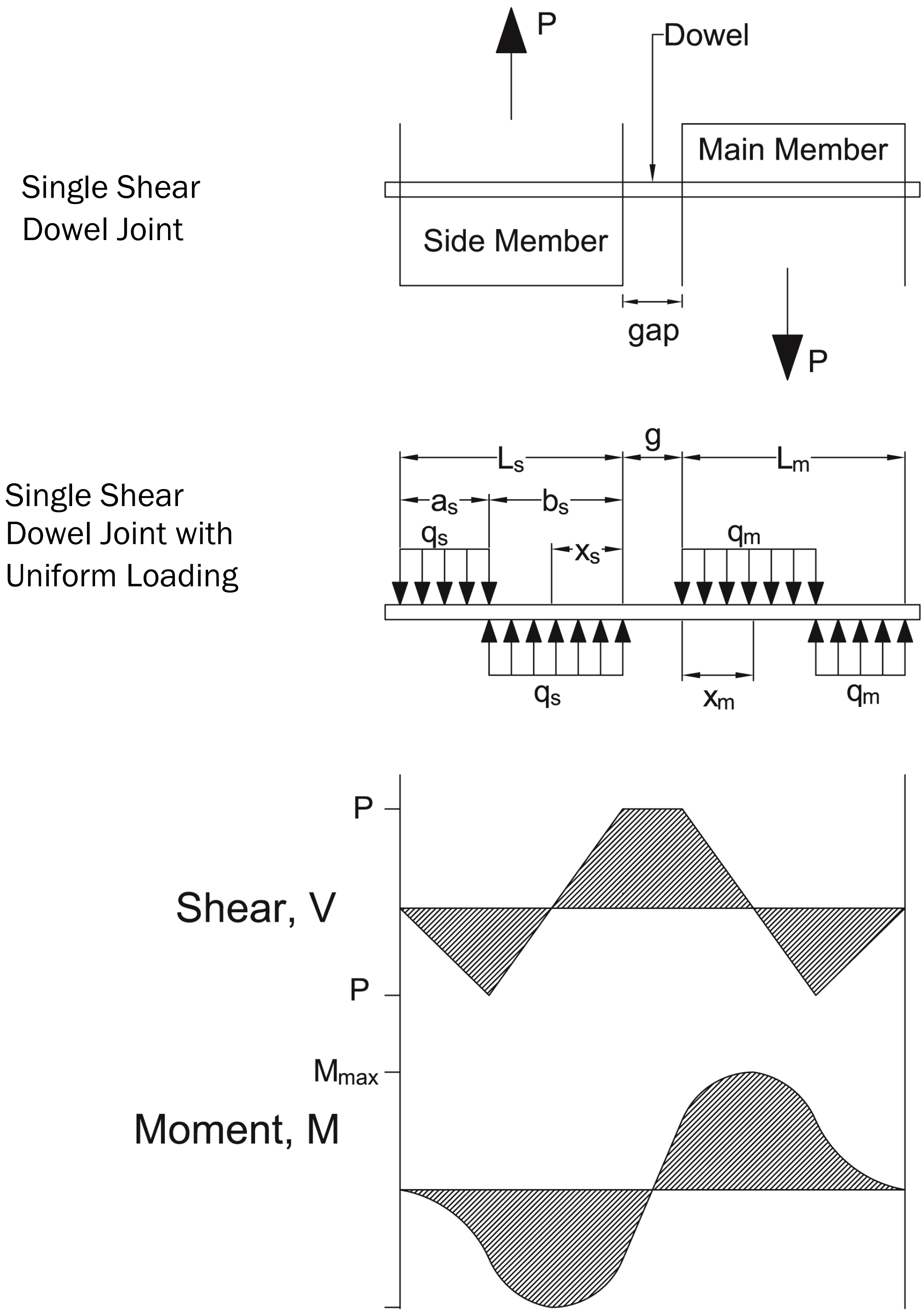


Figure 2-5 Single Shear Connection - Mode II



2.3 Tapered Tip Equation Derivation

Equations considering effect of a tapered fastener tip are based on connections composed entirely of solid-cross section members but with bearing resistance in the penetrated member based on a combination of full diameter bearing and reduced diameter bearing of the tapered fastener tip.

2.3.1 Mode I

The bearing resistance of the fastener with a tapered tip can be calculated as the combination of bearing resistance provided by the full diameter portion of the fastener plus the bearing resistance provided by the tapered tip which is modeled as having diameter vary linearly from full diameter to a point over the length of the tapered tip, E. Maximum load P is determined by:

$$P = q_m(p-E)+\frac{1}{2}q_m(E) \qquad (19)$$

The Mode I_m equation can be further simplified to:

$$P = q_m\left(p - \frac{E}{2}\right) \qquad (20)$$

For single shear connections, bearing in the side member is based on diameter, D. Maximum load P is determined by the characteristic equation:

$$P = q_s L_s \qquad (3)$$

Considering the geometry of a double shear connection, where the tip is in the side member, maximum load, P, is determined by:

$$P = q_m L_m \qquad (2)$$

$$P = q_s\left(2L_s - \frac{E}{2}\right) \qquad (21)$$

2.3.2 Modes II-IV

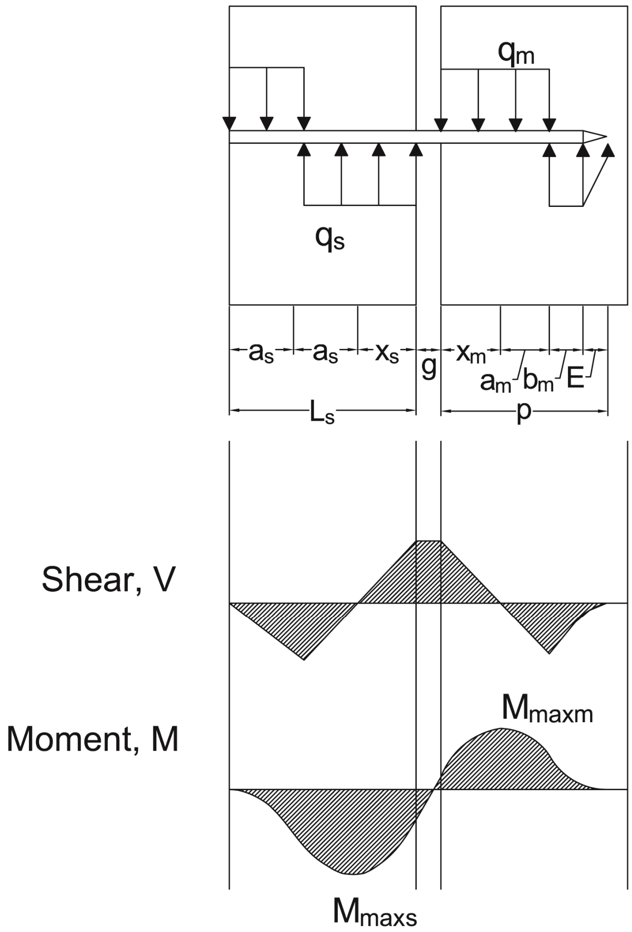
Figure 2-6 illustrates a general dowel bearing scenario with a tapered tip connector. For calculation purposes, let q define member bearing resistance (lb/in.) determined by the equation q = F_eD_b, p define penetration length (in), E define the length of taper (in), and let x represent the location of zero shear (x = p-a-b-E). The resulting maximum moment due to dowel bearing, M_b, equals $qa^2 + qE^2/24$. Recognizing that $a_m = p/2 - x_m/2 - E/4$ and x = P/q results in the following equations for maximum moment due to dowel bearing:

$$M_{bs} = \frac{P^2}{4q_s} - \frac{PL_s}{2} + \frac{q_sL_s^2}{4} \qquad (22)$$

$$M_{bm} = \frac{P^2}{4q_m} - P\left(\frac{p}{2}-\frac{E}{4}\right) + q_m\left(\frac{p^2}{4} - \frac{pE}{4} + \frac{5E^2}{48}\right) \qquad (23)$$

The maximum moment due to dowel bending is limited to the moment provided by the dowel in bending which is represented by a concentrated moment acting at the point of zero shear (x = P/q). Moment resistance of the dowel, M_s and M_m, assuming ideally plastic behavior, is expressed in equations 8 and 9.

Figure 2-6 Dowel Bearing with Tapered Tip Connector



Substitution of values of maximum bearing resistance and maximum moment resistance as applicable for the yield mode into characteristic equation for single shear (equation 10) and for double shear (equation 11) result in equations the maximum load, P, as follows:

Mode II
Single Shear:

$$P^2\left(\frac{1}{4q_s}+\frac{1}{4q_m}\right)+P\left(\frac{L_s}{2}+g+\frac{p}{2}-\frac{E}{4}\right)$$
$$-\left(\frac{q_sL_s^2}{4}+\frac{q_mp^2}{4}-\frac{q_mpE}{4}+\frac{q_m5E^2}{48}\right)=0 \quad (24)$$

Mode III$_m$
Single Shear:

$$P^2\left(\frac{1}{2q_s}+\frac{1}{4q_m}\right)+P\left(g+\frac{p}{2}-\frac{E}{4}\right)$$
$$-\left(M_s+\frac{q_mp^2}{4}-\frac{q_mpE}{4}+\frac{q_m5E^2}{48}\right)=0 \quad (25)$$

Mode III$_s$
Single Shear:

$$P^2\left(\frac{1}{4q_s}+\frac{1}{2q_m}\right)+P\left(\frac{L_s}{2}+g\right)-\left(\frac{q_sL_s^2}{4}+M_{dm}\right)=0 \quad (26)$$

Double Shear:

$$\frac{P^2}{4}\left(\frac{1}{4q_s}+\frac{1}{2q_m}\right)+\frac{P}{2}\left(\frac{L_s}{2}+g\right)-\left(\frac{q_sL_s^2}{4}+M_{dm}\right)=0 \quad (27)$$

Mode IV
Single Shear:

$$P^2\left(\frac{1}{2q_s}+\frac{1}{2q_m}\right)+Pg-\left(M_{ds}+M_{dm}\right)=0 \quad (28)$$

Double Shear:

$$\frac{P^2}{4}\left(\frac{1}{2q_s}+\frac{1}{2q_m}\right)+\frac{Pg}{2}-\left(M_{ds}+M_{dm}\right)=0 \quad (29)$$

For double shear where side member penetrations are unequal, L_s shall be taken as the minimum bearing length in either of the two side members.

2.3.3 NDS Assumption

For connectors with a tapered tip, the NDS specifies that bearing length is permitted to be calculated as L = p – E/2. Example 2.4 in Part 3 of this report shows that the NDS requirement closely approximates results from the more detailed evaluation of a tapered tip on bearing resistance in accordance with the foregoing derivations.

2.4 Equation Derivation for Connections with Members of Hollow Cross Section

Following the approach for connections composed entirely of members having solid cross sections, equations for connections composed of a combination of members having solid and hollow cross section can be developed as shown in Section 2.4 for yield modes depicted in Tables 1-2 and 1-3. Equations in Section 2.4 are based on assumed connection configurations composed entirely of hollow cross section members for both side and main members.

2.4.1 Mode I

Yield modes I_m and I_s model connections limited by uniform bearing in the main and side member(s). Figure 2-7, Case A, shows that the maximum value for shear, P, is determined by the following equations:

$$P = 2q_mt_{wm} \quad (30)$$

$$P = 2q_st_{ws} \quad (31)$$

Similarly, considering the geometry of a double shear connection, the maximum value for shear, P, is determined by the same main member shear equation shown above (30) and an adjusted side member equation:

$$P = 2q_mt_{wm} \quad (30)$$

$$P = 4q_st_{ws} \quad (32)$$

2.4.2 Modes II-IV

For modes II-IV, considering conditions where limiting or maximum moments act on the dowel simplifies equation derivation (see Figure 2-7). In Case B, maximum moment is based on dowel bearing. In Case C, maximum moment is based on dowel bending. Note that maximum moments for both cases occur at points of zero shear. Parsons (2001) showed that for hollow cross section members, the controlling load cases show that all dowel yielding occurs in the wall next to the shear plane, therefore only these cases are considered in this document.

Maximum moment due to dowel bearing, shown in Case B of Figure 2-7, represents a load condition where the dowel is sufficiently large to prevent yielding of the dowel (dowel bending). For calculation purposes, let q define member bearing resistance (lb/in.) ($q = F_eD_b$), t_w define member wall thickness (in.), and let x represent the location of zero shear ($x = 2(t_w - a)$).

The resulting maximum moment due to dowel bearing, M_b, equals $qa^2 + 2qav - qt_wv$. Recognizing that $a = t_w - x/2$ and $x = P/q$, and substituting, results in $M_b = P^2/(4q) - P(t_w + v) + qt_w^2 + qt_wv$ in the member.

Subscripts *s* and *m* indicate side and main member in the following equations for maximum moment due to dowel bearing:

Single Shear:

$$M_{bs} = \frac{P^2}{4q_s} - P(t_{ws} + v_s) + q_s t_{ws}^2 + q_s t_{ws} v_s \qquad (33)$$

$$M_{bm} = \frac{P^2}{4q_m} - P(t_{wm} + v_m) + q_s t_{wm}^2 + q_m t_{wm} v_m \qquad (34)$$

Double Shear:

$$M_{bs} = \frac{P^2}{16q_s} - P\left(\frac{t_{ws}}{2} + \frac{v_s}{2}\right) + q_s t_{ws}^2 + q_s t_{ws} v_s \qquad (35)$$

Maximum moment due to dowel bending is shown in Case C of Figure 2-7. In this case, maximum moment is limited to the moment provided by the dowel in bending which is represented by a concentrated moment acting at the point of zero shear ($x = P/q$). Moment resistance of the dowel, assuming ideally plastic behavior, is expressed in equations 8 and 9.

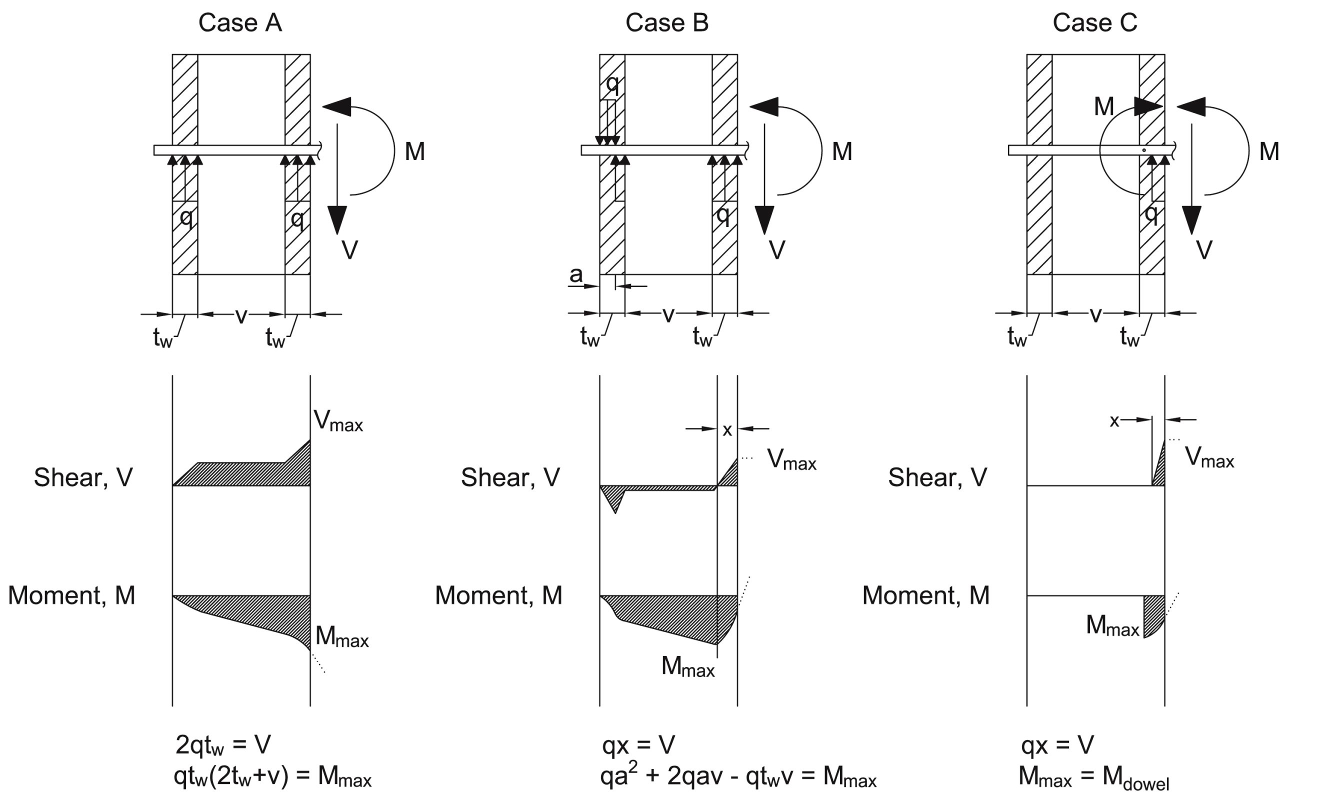
Assumed hollow cross section loading conditions for each of the yield modes I-IV are provided in Figure 2-8. Following the method established in Part 1, the maximum load, P, can be determined using the same characteristic equations used for solid members (equations 10 and 11).

Similar to solid members, yield modes II and III_m are not possible for double shear connections, as the assumed symmetry of double shear connections does not permit these modes to occur.

2.4.3 Mode II

Yield Mode II models a connection limited by dowel bearing in the side and main members. The maximum induced moment was previously determined (equations 33 and 34). Substituting M_{bs} and M_{bm} into the characteristic equation for M_s and M_m results in the following quadratic equation, expressed in terms of known properties for dowel-bearing resistance (q_s and q_m), dowel-bearing length (L_s and L_m), and the gap between members (g), and a single unknown variable, P.

Figure 2-7 General Conditions of Dowel Loading – Hollow Cross Section Members



Single Shear:

$$P^2\left(\frac{1}{4q_s}+\frac{1}{4q_m}\right)+P\left(t_{ws}+v_s+g+t_{wm}+v_m\right) - \left(q_s t_{ws}^2+q_s t_{ws}+q_m t_{wm}^2+q_m t_{wm} v_m\right)=0 \tag{36}$$

2.4.4 Mode III$_m$

Mode III$_m$ models a connection limited by dowel bearing in the main member and dowel bending in the side member. Dowel bending will occur in the member wall closest to the shear plane. Substituting M_{ds} (equation 8) and M_{bm} (equation 34) into the characteristic equation (equations 10 and 11) for M_s and M_m results in the following quadratic equation expressed in terms of known properties for dowel-bearing resistance (q_s and q_m), dowel-bearing length (L_m), dowel moment resistance (M_{ds}) and the gap between members (g), and a single unknown variable, P.

Single Shear:

$$P^2\left(\frac{1}{2q_s}+\frac{1}{4q_m}\right)+P\left(g+t_{wm}+v_m\right) - \left(M_{ds}+q_m t_{wm}^2+q_m t_{wm} v_m\right)=0 \tag{37}$$

2.4.5 Mode III$_s$

Mode III$_s$ models a connection limited by dowel bearing in the side member(s) and dowel bending in the main member. Substituting M_{bs} (equations 33 and 35) and M_{dm} (equation 9) into the characteristic equation for M_s and M_m results in the following quadratic equation(s) expressed in terms of known properties for dowel-bearing resistance (q_s and q_m), dowel-bearing length (L_s), dowel moment resistance (M_{dm}) and the gap between members (g), and a single unknown variable, P.

Single Shear:

$$P^2\left(\frac{1}{4q_s}+\frac{1}{2q_m}\right)+P\left(t_{ws}+v_s+g\right) - \left(q_s t_{ws}^2+q_s t_{ws} v_s+M_{dm}\right)=0 \tag{38}$$

Double Shear:

$$\frac{P^2}{4}\left(\frac{1}{4q_s}+\frac{1}{2q_m}\right)+\frac{P}{2}\left(t_{ws}+v_s+g\right) - \left(q_s t_{ws}^2+q_s t_{ws} v_s+M_{dm}\right)=0 \tag{39}$$

2.4.6 Mode IV

Mode IV models a connection limited by dowel bending in the main and side member(s). Substituting M_{ds} and M_{dm} (equations 8 and 9) into the characteristic equations for M_s and M_m yields the same quadratic equations developed for solid sections (equations 17 and 18).

An alternative, energy based derivation, producing the same results in different equation format, is provided in Parsons (2001).

PART 3. Example Problems

Example 3.1 Bolted Connection with Gap

Problem Statement: Determine the reference design value, Z, for a wood-to-wood connection single shear bolted connection. Compare values for gap distance, g, equal to 0 in., ¼ in., and ½ in. Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

Given: Parallel and perpendicular to grain dowel bearing strengths:

Both side and main member thicknesses are 1.5 in. Each connection uses a single ½ in. diameter bolt (SAE J429 Grade 1).

$L_s = L_m = 1.5$ in.
$D = 0.5$ in.
$F_b = 45{,}000$ psi
$F_{e\parallel} = 4800$ psi, $F_{e\perp} = 2550$ psi
$g = 0, 0.25, 0.5$ in.

Bearing length, in. & grain direction		$P_{5\%}/R_d$, lbs						Z[1], lbs
Main	Side	I_m	I_s	II	III_m	III_s	IV	
				Gap distance, g = 0 in.				
1-½‖	1-½‖	900	900	<u>414</u>	550	550	663	414
1-½‖	1-½⊥	720	383	<u>250</u>	380	324	442	250
1-½⊥	1-½⊥	383	383	<u>176</u>	289	289	387	176
				Gap distance, g = ¼ in.				
1-½‖	1-½‖	900	900	<u>370</u>	482	482	576	370
1-½‖	1-½⊥	720	383	<u>224</u>	341	284	393	224
1-½⊥	1-½⊥	383	383	<u>157</u>	258	258	349	157
				Gap distance, g = ½ in.				
1-½‖	1-½‖	900	900	<u>333</u>	426	426	501	333
1-½‖	1-½⊥	720	383	<u>202</u>	307	250	350	202
1-½⊥	1-½⊥	383	383	<u>142</u>	231	231	315	142

[1]Connection values have not been adjusted for end use conditions such as load duration, wet service, and temperature.

Example 3.2 Lag Screw Connection

Problem Statement: Determine the reference design value for a wood-to-wood single shear lag screw connection. Compare values for different connection conditions described in Cases 1–4 below. The main member is loaded parallel to grain for all cases. Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

Note: *This example illustrates the application of TR12 equations to account for varying bearing and moment resistances of lag screws in a wood-to-wood connection. A similar approach can be applied to other threaded fasteners such as bolts and wood screws by accounting for specific attributes (diameter, root diameter, dowel bending strength) of those fastener types.*

Given Data:

Parallel and perpendicular to grain dowel bearing strengths:

$F_{e\parallel} = 5600$ psi, $F_{e\perp} = 3650$ psi

Dowel bending strength:

$F_b = 45{,}000$ psi

Side member thickness:

$L_s = 1.5$ in.

Lag screw bearing length in the main member:

$L_m = 3$ in.

Lag screw diameters:

$D = 0.375$ in.,

$D_r = 0.265$ in.

Gap distance between side and main member:

$g = 0$ in.

Case 1 (Figure 3-2A): Moment resistance, side member bearing, and main member bearing are based on fastener root diameter, D_r. ***Note:*** *These are the assumptions used to develop the NDS tabular values.*

Side Member:

$$q_s = F_{es}D_r$$

$$M_s = \frac{F_b D_r^3}{6}$$

Main Member:

$$q_m = F_{em}D_r$$

$$M_m = \frac{F_b D_r^3}{6}$$

Case 2 (Figure 3-2B): Moment resistance and side member bearing are based on fastener root diameter, D_r. Bearing resistance in the main member is based on D.

Side Member:

$$q_s = F_{es}D_r$$

$$M_s = \frac{F_b D_r^3}{6}$$

Main Member:

$$q_m = F_{em}D$$

$$M_m = \frac{F_b D_r^3}{6}$$

Case 3 (Figure 3-2C): Moment resistance is based on fastener root diameter, D_r, and bearing resistance is based on D.

Side Member:

$$q_s = F_{es}D$$

$$M_s = \frac{F_b D_r^3}{6}$$

Main Member:

$$q_m = F_{em}D$$

$$M_m = \frac{F_b D_r^3}{6}$$

Case 4 (Figure 3-2D): Fastener moment resistance and bearing resistance are based on diameter, D (e.g. connection with unthreaded shank extending deep into the main member). Additional calculations will determine the required length of unthreaded shank penetration into the main member.

Note: *Where unthreaded shank penetration into the main member is not adequate to enable development of the moment resistance of the fastener based on diameter D, the fastener rood diameter, D_r, is the appropriate diameter for calculation of fastener moment resistance in the main member.*

Side Member:

$$qs = F_{es}D$$

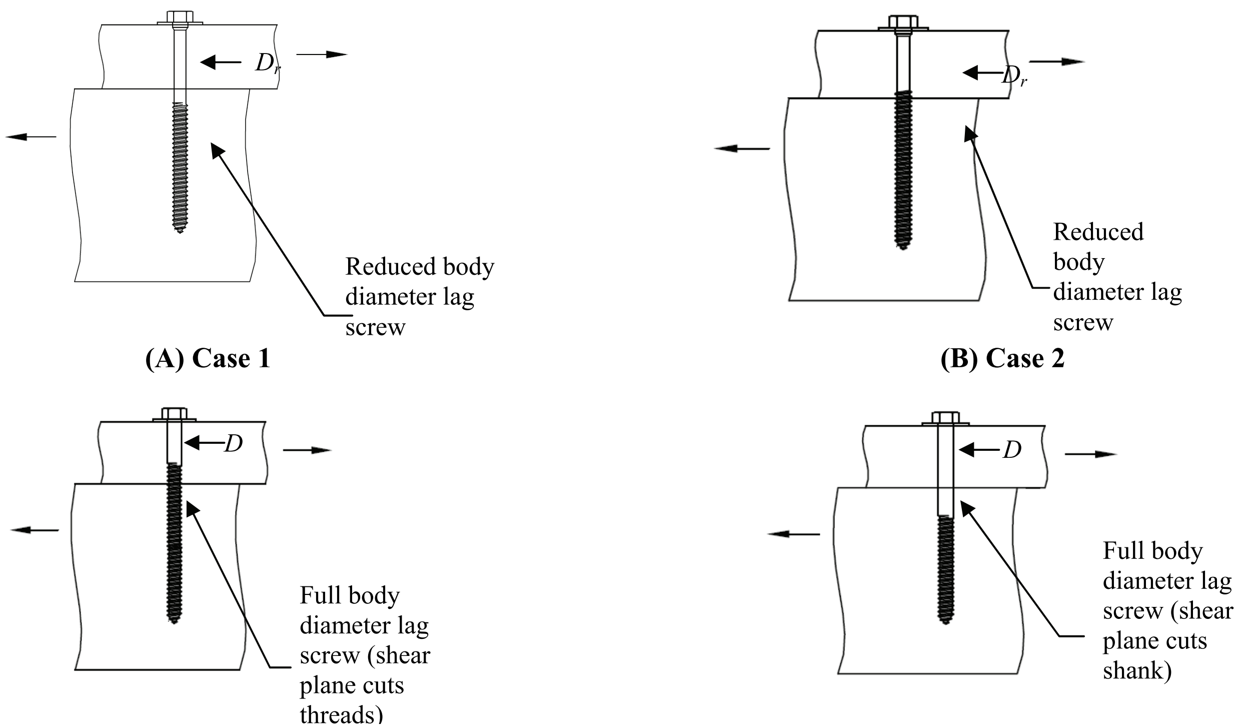
$$M_s = \frac{F_b D^3}{6}$$

Main Member:

$$q_m = F_{em}D$$

$$M_m = \frac{F_b D^3}{6}$$

Figure 3-2



(A) Case 1 **(B) Case 2**

(C) Case 3 **(D) Case 4**

Bearing length, in. & grain direction			$P_{5\%}/R_d$, lbs				Z[1], lbs
Side	I_m	I_s	II	III_m	III_s	IV	
			Case 1 (NDS Tabular Basis)[2]				
1- ½∥	1113	557	420	478	260	201	201
1- ½⊥	890	290	304	353	153	143	143
			Case 2				
1- ½∥	1575	557	548	629	275	218	218
1- ½⊥	1260	290	400	457	160	152	152
			Case 3				
1- ½∥	1575	788	595	671	357	239	239
1- ½⊥	1260	411	431	495	207	170	170
			Case 4				
1- ½∥	1575	788	595	697	406	403	403
1- ½⊥	1260	411	431	513	249	286	286

[1]Connection values have not been adjusted for end use conditions such as load duration, wet service, and temperature.
[2]Case 1 represents assumptions used to develop NDS tabular values. Since the NDS equations utilize only a single diameter in calculation of reference design value, the least diameter (e.g. D_r) is conservatively used to estimate moment and bearing resistances. In this example, Cases 2-4 demonstrate that more efficient designs can be achieved by considering the varying moment and bearing resistances of the fastener in lieu of the least diameter assumption.

Example 3.3 Lag Screw Penetration

Problem Statement: Determine the minimum required length of shank penetration in the main member to ensure that fastener moment resistance in the main member is limited by diameter, D, of the shank rather than root diameter, D_r, at the threads. Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

Note: *This example utilizes a lag screw connection to illustrate an approach for determining minimum required length of shank penetration to ensure fastener moment resistance in the main member is limited by diameter, D. A similar approach can be applied to other threaded fasteners such as bolts and wood screws by accounting for specific dimensions (diameter, root diameter, dowel bending strength) of those fasteners.*

Given: The 3/8" diameter lag screw connection in Figure 3.3A has a reference design value of 403 lbs (Mode IV) based on diameter, D (see Case 4 of Example 3.2).

P = (Reference design value)(R_d)
= (403 lbs)(3.2) = 1289.6 lbs

$q_m = F_{em}D$
= (5600psi)(0.375 in.) = 2100 lb/in.

Fastener moment resistance at shank:

$$D = \frac{F_{b,5\%}D^3}{6} = 395.5 \text{ in.-lbs}$$

Fastener moment resistance at root:

$$D_r = \frac{F_{b,5\%}D_r^3}{6} = 139.6 \text{ in.-lbs}$$

Solution:

The basic dowel loading condition for Mode IV is shown in Figure 3.3B. Since the reference value is based on Mode IV yield, the maximum moment is equal to the moment resistance of the dowel (e.g. M_{max} = 395.5 in.-lbs based on the moment resistance provided by shank diameter, D). Zero shear and maximum moment occur at distance, x_m, from the shear plane:

$$x_m = \frac{P}{q_m} = \frac{1289.6}{2100} = 0.61 \text{ in.}$$

To estimate moment beyond x_m an alternate loading condition can be constructed for the dowel as shown in Figure 3.3C. The length of bearing needed to develop the maximum moment is calculated as:

$$2a = 2\left(\frac{M_{max}}{q_m}\right)^{0.5} = 2\left(\frac{395.5}{2100}\right)^{0.5} = 0.87 \textit{ in.}$$

The fastener moment diagram in the main member from $0 \le x_1 \le 2a$ can be constructed from the alternate load condition in Figure 2.3C.

For $0 < x_1 \le a$:

$$M = M_{max} - \frac{q_m x_1^2}{2}$$

Substituting $M_{reduced}$ for M, distance x_1 from the point of maximum moment to the point of reduced moment can be calculated as:

$$x_1 = \left(\frac{2(M_{max} - M_{reduced})}{q_m}\right)^{0.5}$$

Distance x_2 from the shear plane to the point of reduced moment can be calculated as:

$$x_2 = \frac{P}{q_m} + \left(\frac{2(M_{max} - M_{reduced})}{q_m}\right)^{0.5}$$

For $a < x_1 \le 2a$

$$M = M_{max} - \frac{q_m a^2}{2} - \left(q_m a - \frac{q_m x_1}{2}\right)x_1$$

Substituting $M_{reduced}$ for M, distance x_1 from the point of maximum moment to the point of reduced moment can be calculated as:

$$x_1 = 2a - \left(\frac{2M_{reduced}}{q_m}\right)^{0.5}$$

Distance x_2 from the shear plane to the point of reduced moment can be calculated as:

$$x_2 = \frac{P}{q_m} + 2a - \left(\frac{2M_{reduced}}{q_m}\right)^{0.5}$$

<u>Minimum penetration length</u>

The value of $M_{reduced}$ is taken as 139.6 in.-lbs which equals the moment resistance at the root diameter, D_r. For $M_{reduced}$ = 139.6 in.-lbs, $a < x_1 \le 2a$ and the minimum required shank penetration, x_2, equals 1.12 in.

Figure 3-3A

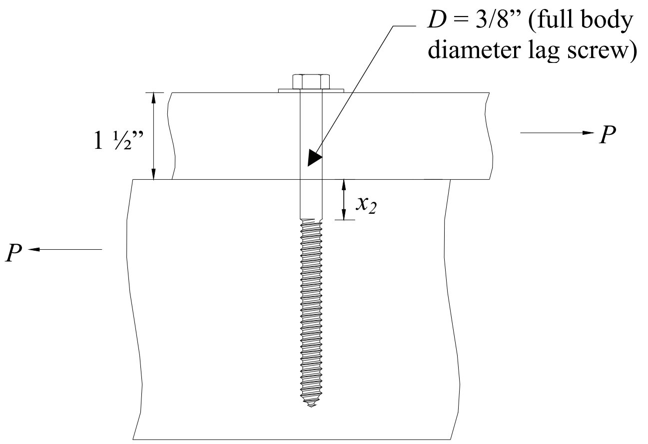


Figure 3-3B

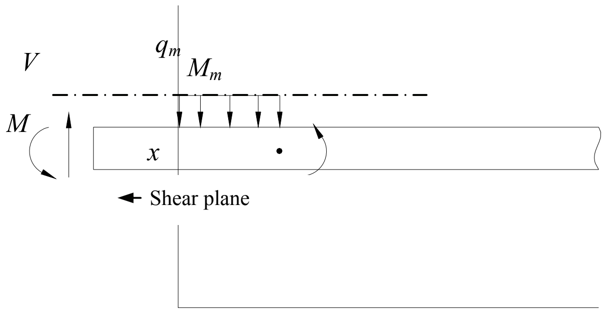
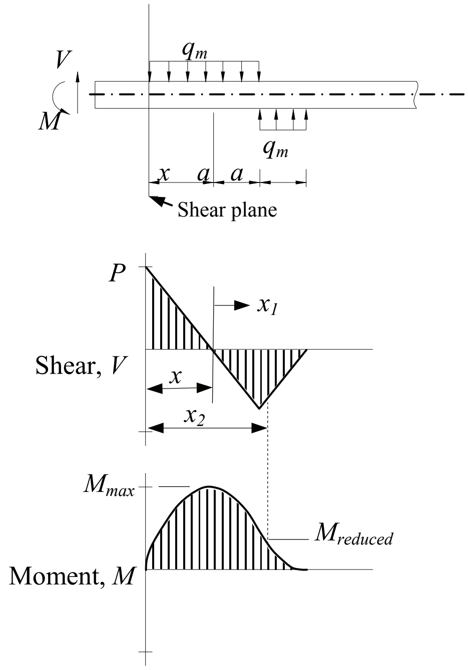


Figure 3-3C



Example 3.4 Steel Side Member to Wood Main Member – Tapered Tip Effects

Problem Statement: Compare reference design values, Z, for a single shear nail connection at 12D, 10D, 8D, 6D, and 4D penetrations using the following assumptions:

For "EYM tip derivation" case:

$L_m = p$

Tapered tip equations 24 through 29 in accordance with this report

For "NDS" case:

$L_m = p - E/2$

General dowel equations in Table 1-1 of this report
Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

Given: Dowel bearing strength is 4700 psi. Side member is 16 gage ASTM A653, Grade 33 steel. Nail diameter is 0.131" and is of adequate length to accommodate the given penetrations. Tapered tip length, E, is equal to 2D. Main member is loaded parallel to grain.

L_s = 0.06 in.

D = 0.131 in.

E = 0.262 in.

F_{em} = 4700 psi

F_{es} = 61,850 psi

F_b = 100,000 psi

g = 0 in.

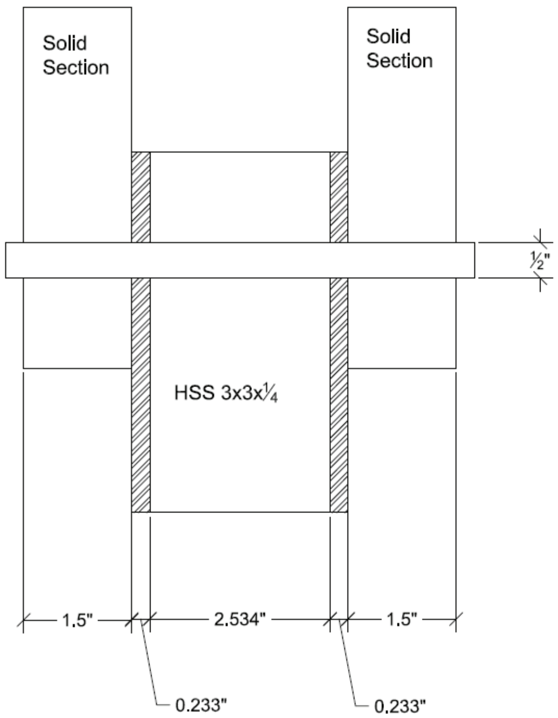
Penetration Depth (p)	EYM Tip derivation Z[1] (lbs)	EYM Tip derivation Controlling Mode	NDS (L_m = p-E/2) Z[1] (lbs)	NDS (L_m = p-E/2) Controlling Mode
12D (1.57")	97	III_s	97	III_s
10D (1.31")	97	III_s	97	III_s
8D (1.05")	97	III_s	97	III_s
6D (0.79")	79	II	78	II

[1]Connection values have not been adjusted for end use conditions such as load duration, wet service, and temperature.

Example 3.5 Bolted Connection through Solid and Hollow Members – Double Shear

Problem Statement: Using equation in Table 1-3, determine the reference design value, Z, for a double shear bolted connection with a hollow section main member and solid cross section side members shown in Figure 3.5. Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

Figure 3-5



$q_m = F_{em}D = 87{,}000$ psi(0.5 in.) = 43,500 lb/in.

$q_s = F_{es}D = 4800$ psi(0.5 in.) = 2400 lb/in.

$$M_m = M_s = \frac{F_b D^3}{6} = \frac{45{,}000\,\text{psi}(0.5\,\text{in.})^3}{6} = 937.5 \text{ in.-lb}$$

$P_{5\%}/R_d$, lbs				Z^1, lbs
I_m	I_s	III_s	IV	
5068	1800	1413	1825	1413

[1]Connection values have not been adjusted for end use conditions such as load duration, wet service, and temperature.

Given: Main member is an HSS 3x3x¼, side members are 1.5" thick. A single ½" diameter SAE J429 Grade 1 bolt is used to make the connection.

t_{wm} = 0.233 in.

v_m = 2.534 in.

L_s = 1.5 in.

D = 0.5 in.

F_{em} = 87,000 psi

F_{es} = 4800 psi

F_b = 45,000 psi

g = 0 in.

References

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American Wood Council

AWC Mission Statement

To increase the use of wood by assuring the broad regulatory acceptance of wood products, developing design tools and guidelines for wood construction, and influencing the development of public policies affecting the use and manufacture of wood products.

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